Introduction to Geostatistics for Resources Evaluation and Estimation

PREPARED BY:
DR. MOHAMAD NUR HERIAWAN
DEPARTMENT OF MINING ENGINEERING – ITB

Jurusan Teknik Pertambangan
Fakultas Teknik
Universitas Negeri Padang
Padang, 1 April 2013
Lecturer: **Mohamad Nur Heriawan**

Associate Professor at Department of Mining Engineering, Faculty of Mining and Petroleum Engineering, Institute of Technology Bandung (ITB)

Educational Background:

- 1993 – 1998: Bachelor Degree at Department of Mining Engineering – ITB, Option: Mining Exploration
- 1998 – 2000: Master Degree at Department of Applied Geophysics – ITB
- 2002 – 2003: Postgraduate Diploma at Centre of Geostatistics, Paris School of Mines, France
- 2004 – 2007: Doctoral Course at Department of Environmental Science, Kumamoto University, Japan
Outlines

1. WHY GEOSTATISTICS?
2. INTRODUCTION TO BASIC LINEAR GEOSTATISTICS
3. GEOSTATISTICS FOR RESOURCE ESTIMATION
Why Geostatistics?
Since introduced by D. Krige (1955) and G. Matheron (1960), geostatistics have been widely used in mining industry for resource and reserve estimation.

Geostatistical analysis provides a powerful tool for enhancing the prediction and decision making capabilities of mine planners and geologists.

The experimental semivariogram provides the only measure of whether a deposit, or part of a deposit, is best analyzed using geostatistical methods or whether classical statistics would suffice.

Geostatistics provides the best possible weighting for samples used in reserve estimation so as to produce the lowest possible error of estimation.

Computerized simulation of in situ coal quality for improving predictions of the variability of coal qualities have been identified as the most significant parameters affecting coal supply (Whateley, 2002).
Review on Basic Statistics

1. Univariate Statistics

Analysis on single variable without considering their location. The data is assumed to be a random variable.

2. Bivariate Statistics

Analysis on two different variables located in the same location.

3. Spatial Statistics

Analysis on a variable with considering the spatial aspect of data. It can be applied for natural phenomena, by assuming that the data is a random function.
Parameters for Measure the Central Tendency

1. Mean : \( \mu = \frac{1}{n} \sum_{i=1}^{n} x_i \)

2. Median : central data value

3. Mode : highest frequency value

4. Skewness : 
\[
Skewness = \frac{\frac{1}{n} \sum_{i=1}^{n} (x_i - \mu)^3}{\sigma^3}
\]

5. Kurtosis : 
\[
Kurtosis = \frac{\sum_{i=1}^{n} (x_i - \mu)^4}{\sigma^4}
\]
Skewness of some histograms: a) symmetric (normal distribution); b) negative skewness; and c) positive skewness (lognormal distribution)
Parameters for Measure the Dispersion

1. Range : \( range = X_{max} - X_{min} \)

2. Variance : \[ \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \mu)^2}{n-1} \]

3. Standard Deviation : \[ \sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \mu)^2}{n-1}} \]

4. Coefficient of Variation : \( CV = \frac{\sigma}{\mu} \)
<table>
<thead>
<tr>
<th>Type of mineral deposits</th>
<th>CV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gold: California, USA; placer Tertiary</td>
<td>5.10</td>
</tr>
<tr>
<td>Tin: Pemali, Bangka, Indonesia; Primary</td>
<td>2.89</td>
</tr>
<tr>
<td>Gold: Loraine, South Africa; Black Bar</td>
<td>2.81</td>
</tr>
<tr>
<td>Gold: Norseman, Australia; Princess Royal Reef</td>
<td>2.22</td>
</tr>
<tr>
<td>Gold: Grasberg, Papua, Indonesia</td>
<td>2.01</td>
</tr>
<tr>
<td>Lead: Grasberg, Papua, Indonesia</td>
<td>1.57</td>
</tr>
<tr>
<td>Tungsten: Alaska</td>
<td>1.56</td>
</tr>
<tr>
<td>Gold: Shamva, Rhodesia</td>
<td>1.55</td>
</tr>
<tr>
<td>Uranium: Yeelirrie, Australia</td>
<td>1.19</td>
</tr>
<tr>
<td>Gold: Vaal Reefs, South Africa</td>
<td>1.02</td>
</tr>
<tr>
<td>Zinc: Grasberg, Papua, Indonesia</td>
<td>0.87</td>
</tr>
<tr>
<td>Zinc: Frisco, Mexico</td>
<td>0.85</td>
</tr>
<tr>
<td>Gold: Loraine, South Africa; Basalt Reef</td>
<td>0.80</td>
</tr>
<tr>
<td>Nickel: Kambalda Australia</td>
<td>0.74</td>
</tr>
<tr>
<td>Copper</td>
<td>0.70</td>
</tr>
<tr>
<td>Manganese</td>
<td>0.58</td>
</tr>
<tr>
<td>Lead: Frisko, Mexico</td>
<td>0.57</td>
</tr>
<tr>
<td>Sulphur in Coal: Lati Mine, Berau, Indonesia</td>
<td>0.48</td>
</tr>
<tr>
<td>Lateritic Nickel: Gee Island, East Halmahera, Indonesia</td>
<td>0.44</td>
</tr>
<tr>
<td>Iron ore</td>
<td>0.27</td>
</tr>
<tr>
<td>Bauxite</td>
<td>0.22</td>
</tr>
</tbody>
</table>
Scatter plot $\rightarrow$ used to plot the correlation between two different variables (i.e. $x$ and $y$ variables) located in the same position

![Scatter plots with different correlation coefficients](image-url)
Covariance \( (C_{xy}) \rightarrow \) used to measure the dispersion of two different variables (i.e. \( x \) and \( y \) variables) located in the same position:

\[
C_{xy} = \frac{1}{n} \sum_{i=1}^{n} (x_i - \mu_x)(y_i - \mu_y)
\]

Coefficient of correlation \( (\rho) \rightarrow \) used to measure the correlation between two different variables (i.e. \( x \) and \( y \) variables) located in the same position:

\[
\rho = \frac{\frac{1}{n} \sum_{i=1}^{n} (x_i - \mu_x)(y_i - \mu_y)}{\sigma_x \sigma_y}
\]

Linear regression of two variables:

\[
Y = aX + b
\]

\[
a = \rho \frac{\sigma_x}{\sigma_y}
\]

where: \( a = \text{slope} \)

\[
b = \mu_y - a\mu_x
\]

\( b = \text{Y-intercept} \)
About Outlier…

- Outlier can be very sensitive to the data distribution and spatial structure in estimation (i.e. tend to generate overestimate).

- There is no strict solution to decide how to handle outliers, or even decide what an outlier is → any solution is based on feelings and common sense.

- The presence of outliers may require a special robust estimator of the mean, i.e. “Sichel’s-t-estimator” (Sichel, 1966).

- At this topic we only discuss the problem of correcting individual values in practice → by cutting/capping high values.

- Other solution → the distribution of data larger and smaller than twice standard deviation can be considered as outliers (anomaly).
Cumulative frequency curve of uranium grades and suggested correction for outliers (David, 1980).
Probability plot of: (a) Pb and (b) Zn grades for each rock types. The parts noted by dotted circles show some lowest and highest values which are considered as outliers data, while the horizontal dotted line in the graph is cut-off for lower and higher grades in intrusive group (Heriawan et al., 2008).
The perspective views of Pb and Zn grades in intrusive group for: (a) Pb and (b) Zn with blue and grey colors show the high grade and low grade respectively.

Statistics of Pb and Zn grades for each cut-off in intrusive group

<table>
<thead>
<tr>
<th>Rocktype</th>
<th>Pb &gt;0.005%</th>
<th>Pb &lt;0.005%</th>
<th>Zn &gt;0.01%</th>
<th>Zn &lt;0.01%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nb of values</td>
<td>979</td>
<td>10671</td>
<td>2666</td>
<td>9107</td>
</tr>
<tr>
<td>Min</td>
<td>0.0050</td>
<td>0.0003</td>
<td>0.0100</td>
<td>0.0004</td>
</tr>
<tr>
<td>Max</td>
<td>0.8537</td>
<td>0.0049</td>
<td>3.2509</td>
<td>0.0099</td>
</tr>
<tr>
<td>Mean</td>
<td>0.0354</td>
<td>0.0014</td>
<td>0.0653</td>
<td>0.0050</td>
</tr>
<tr>
<td>Median</td>
<td>0.0086</td>
<td>0.0012</td>
<td>0.0150</td>
<td>0.0047</td>
</tr>
<tr>
<td>Std error</td>
<td>0.0032</td>
<td>0.000010</td>
<td>0.0047</td>
<td>0.000022</td>
</tr>
<tr>
<td>Variance</td>
<td>0.0103</td>
<td>0.000001</td>
<td>0.0579</td>
<td>0.000004</td>
</tr>
<tr>
<td>Coef. of var</td>
<td>2.8744</td>
<td>0.7010</td>
<td>3.6828</td>
<td>0.4128</td>
</tr>
</tbody>
</table>

Source: Heriawan et al., 2008)
Probability plot of Tin grade

Top-cut for Sn grade = 3.26 kg/m³

TDH (kg/m³)
Method to differentiate the background and anomaly data

![Graph showing standard deviation and anomaly levels]

- M-2SD (0.7)
- M-1SD (0.9)
- Mean (1.1)
- M+1SD (1.3)
- M+2SD (1.6)

- Slightly Anomalous
- Anomalous
Recognizing the different population…

Population for low grade

Population for high grade
different population?

Source: Heriawan & Koike, 2008a)
Data-point locations of the different population of sodium contents with the low content is smaller than 1 % (×) and the high content is larger than 1 % (∗)
Fe vs. Ni grades in Laterite Nickel Deposit

- Limonite zone
- Saprolite zone
Classical Statistics vs. Spatial Statistics

- It has been known two methods for statistical analysis of mineral deposit characteristics: classical statistics and spatial statistics.

- Classical statistical is used to define the properties of sample values with assumption that they are realization of random variables.

- In this case, the samples composition/support is relatively ignored, then assumed that all sample values have the same probability to be picked up.

- The presence of trends and ore shoots in mineralization zones is ignored.

- The fact in earth sciences shows that two samples taken in vicinity gives the similar value compared to the others in further distance.
• On the other hand, spatial statistics assumes that the sample values are realizations of random function.

• In this hypothesis, sample values is function of their locations in deposit, then their relative position is considered in analysis.

• The similarity of sample values which is function of the samples distance is the basics theory in spatial statistics.

• In order to define how closely the spatial correlation among points in deposit, we must know the structural function which is represented by variogram (semi-variogram).
Why Spatial Statistics?

- Statistical description has not taken the data location into account.
- Statistical description has not taken the data density into account.
- Statistical description will produce the same result even though the data location is changed randomly.
- Spatial analysis can be prepared by plotting the data distribution (into a map).
Prof. Matheron (1950): Geostatistics has been defined as "the application of probabilistic methods to regionalized variables", which designate any function displayed in a real space.
BOX No 1: Variables that can be modelled by random functions.

- Metal grades, for precious metals, uranium, base metals, coal, diamonds, beach sands, industrial minerals,
- Quality parameters e.g. for iron ore, silica, alumina, loss on ignition and sometimes manganese; for gold, arsenic; for coal, calorific value, ash & sulphur content; for cement, iron content, magnesium oxide, moisture,
- Topographic variables such as seam thickness, overburden thickness, depth to a geological horizon, position of the sea floor,
- Rock type indicators e.g. for distinguishing between sandstone and shale in oil reservoirs, or between different facies in general,
- Porosity and permeability, for both oil reservoirs and aquifers, hydraulic head and transmissivity in hydrology,
- Geochemical trace element concentrations in soil samples and stream sediments,
- Pollutant concentrations in soil & water and in the atmosphere,
- For soil science, trace element concentrations (e.g. Cu & Co), nematode counts in soil,
- In fishery science, fish & egg counts, water temperature, salinity; density of shellfish per unit area,
- In hydrology, rainfall and runoff measurements,
- Tree density in tropical forests.

(Source: Armstrong, 1998)
Application of Geostatistics in Mining

1. Estimating the total reserves
2. Error estimates
3. Optimal sample (or drilling) spacing
4. Estimating block reserves
5. Gridding and contour mapping
6. Simulating a deposit to evaluate a proposed mine plan
7. Estimating the recovery
Fig 2.1. Diagrammatic representation of sulphur grades and a blow-up of the central section. Over the whole 8km length, the sulphur content is clearly not stationary because of the increase in the average. But over shorter sections it can be considered as being locally stationary because the fluctuations dominate the trend.
Fig 2.2. Whereas the variogram starts from zero and rises up to a limit, the spatial covariance starts out from the variance and decreases.

(Source: Armstrong, 1998)
Review of Geostatistics in Coal Industry

- Armstrong (1989): coal industry has shown little interest in using geostatistics compared to metallic mines, because of two factors:

  1. The problems of estimation coal reserve are of secondary importance compare to those of predicting continuity of the seam.

  2. Traditional method obtained the reasonably accurate estimates, so the coal companies felt no need for more sophisticated techniques.

- But, when the coal quality factors are concerned, coal companies take a close look at their estimation procedures.

- The traditional reserve estimation methods give reasonably good predictions of the insitu tonnage, but they are not accurate enough for predicting quality variables on a short term basis, nor do they give any estimate of how accurate their predictions are.
• Except for sulphur content which is often erratic, the distribution of coal variables are slightly and so there are relatively few problems calculating and interpreting the experimental variogram.

• The geostatistical characteristics of coal may be constant within one area, but they vary from area to area, as for example the figures above are seam thickness variogram for: (a) Bowen Basin, (b) Barito Basin, and (c) Tarakan Basin.
Wood (1976) and Sabourin (1975) examined that kriging works in practice for coal as well as for metal, because kriging estimate was consistently closer to the actual figures.

Armstrong (1983) used the geostatistics to optimize the drilling grids of coal data by calculating the estimation variance as a function of the drillhole spacing and then find the spacing that just gives the required precision.

Geostatistical conditional simulations can be used to produce a numerical model of the deposit which duplicates the statistical characteristics of the coal insitu.

Downstream of geostatistics and conditional simulations can help mine planners to predict the characteristics of coal coming out of the pit.
Introduction to Basic Linear Geostatistics
Experimental variogram:

\[ \gamma(h) = \frac{\sum_{i=1}^{N} [z(x_i) - z(x_{i+h})]^2}{2N(h)} \]

where: 
- \( \gamma(h) \) = variogram for particular direction of distance \( h \)
- \( h \) = 1\( d \), 2\( d \), 3\( d \), 4\( d \) (\( d \) = average spacing of data points)
- \( z(x_i) \) = data value on point \( x_i \)
- \( z(x_{i+h}) \) = data value on point separated by \( h \) from \( x_i \)
- \( N(h) \) = number of data pairs

As for example, Au grade (ppm) is known along the quartz vein with sampling spacing (\( d \)) of 2 m in regular:
grade: 7 9 8 10 9 11 11 13 11 12 16 12 10 11 10 12 15 ppm
location: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17
Variogram values in each lag distance:

\[ \gamma(2) = \frac{(7 - 9)^2 + (9 - 8)^2 + (8 - 10)^2 + (10 - 9)^2 + \ldots + (10 - 12)^2 + (12 - 15)^2}{2 \times 16} \text{ ppm}^2 \]

\[ = \frac{(4+1+4+1+4+0+4+4+1+16+16+4+1+1+4+9)}{2 \times 16} = 74/32 = 2.31 \text{ ppm}^2 \]

\[ \gamma(4) = \frac{(1+1+1+4+4+0+1+25+0+36+1+0+1+25)}{2 \times 15} = 101/30 = 3.36 \text{ ppm}^2 \]

\[ \gamma(6) = \frac{(9+0+9+1+16+0+1+9+1+4+25+4+4+16)}{2 \times 14} = 99/28 = 3.54 \text{ ppm}^2 \]

\[ \gamma(8) = \frac{(4+4+9+9+4+1+25+1+1+1+25+0+16)}{2 \times 13} = 100/26 = 3.85 \text{ ppm}^2 \]

\[ \gamma(10) = \frac{(16+4+25+1+9+25+1+9+0+4+16+9)}{2 \times 12} = 119/24 = 4.96 \text{ ppm}^2 \]

\[ \gamma(12) = \frac{(16+16+9+4+49+1+1+4+1+0+1)}{2 \times 11} = 102/22 = 4.64 \text{ ppm}^2 \]

\[ \gamma(14) = \frac{(25+4+16+25+9+1+0+9+1+9)}{2 \times 10} = 99/20 = 4.95 \text{ ppm}^2 \]

\[ \gamma(16) = \frac{(16+9+64+4+1+0+1+1+16)}{2 \times 9} = 112/18 = 6.22 \text{ ppm}^2 \]

\[ \gamma(18) = \frac{(25+49+16+0+4+1+1+4)}{2 \times 8} = 100/16 = 6.25 \text{ ppm}^2 \]

\[ \gamma(20) = \frac{(81+9+4+1+1+1+16)}{2 \times 7} = 113/14 = 8.07 \text{ ppm}^2 \]

\[ \gamma(22) = \frac{(25+1+9+0+9+16)}{2 \times 6} = 60/12 = 5.00 \text{ ppm}^2 \]

\[ \gamma(24) = \frac{(9+4+4+4+36)}{2 \times 5} = 57/10 = 5.70 \text{ ppm}^2 \]
Experimental variogram and population variance (horizontal dotted line shows value of 5.25 ppm²)

Direction of variogram ($\theta$), searching area with angle tolerance ($\theta \pm \alpha/2$) and distance tolerance ($h \pm \Delta h$), modified from David (1977)
Fig. 4.2. Flowchart showing how to calculate experimental variograms

(Source: Armstrong, 1998)
1. Behaviour of Variograms near the Origin

The continuity of variable distribution is related to behaviour of variogram near the origin.

Parabolic behaviour near the origin indicates a high continuity variable, i.e. data with regular distribution: geophysics or geochemical variable, water table, or sometimes coal thickness data.

Linier behaviour near the origin indicates a moderate continuity variable. This type of variogram is commonly valid for ore grades.

Variable with highly erratic give a variogram started by an offset. This uncontinuity is named ”nugget effect” and the value is named nugget variance.
2. Area of Influence (Range)

Commonly $\gamma(h)$ will be increasing while $h$ is increasing, it means that the magnitude of the difference of values between two points is dependant on the distance of both.

The increasing of $\gamma(h)$ is still occurred as long as the values among points are still influenced each others, the area of them is named area of influence. $\gamma(h)$ will be constant up to a value of $\gamma(\infty) = C$ (sill) which actually is population variance (variance a priori).

The area of influence has notation $a$ (range). Beyond this distance, the average of variation of values $Z(x)$ and $Z(x+h)$ are not dependant anymore, or $Z(x)$ and $Z(x+h)$ are not correlated each others. Range $a$ is a measure of area of influence.
An example of variogram of the thickness of sedimentary deposit shows geostatistical parameters: C (sill) and a (range).
3. Nested Structures

If an ore deposit has some different structures (scales), then each of them will give variograms with different range \((a)\) (a measure on dimension of structures). This influence of structures will be overlaid, so they will give a combined variogram which the components can be decomposed.

Variogram with nested structure is commonly occurred if the distance between samples is very close compared to the range \(a\). These variograms were usually occurred on fluviatile deposit with lens or fingering shaped.

![Theoretical variogram with nested structure](image)
4. Nugget Variance and Micro Structure

Variogram with nested structure is commonly occurred if the distance of samples is very close compared to the range $a$. In case the lag distance is chosen largerly then the origin of variogram could not be recorded well, therefore the curve extrapolation toward $h = 0$ will not give $\gamma(0) = 0$, but $\gamma(0) = C_0$ which is known as nugget variance.

The influence of micro structure for choosing the lag distance can be seen by the presence or absence of nugget variance. The nugget effect can be minimized by reducing the lag distance $h$. The presence of nugget variance also could be caused by the sampling error, error in grades analyzes, etc.

Theoretical variogram with nugget variance and micro structure
5. Anisotropy

Due to $h$ is a vector, then a variogram must be defined for any distances. An observation for the change of $\gamma(h)$ according to their direction is possible to generate an anisotropy.

Isotropy
If the variogram for any distances are the same, then $\gamma(h)$ is a function of the absolute value of vector $\bar{h}$ where: $|h| = \sqrt{h_1^2 + h_2^2 + h_3^2}$, if $h_1$, $h_2$, and $h_3$ are components of vector $h$.

Geometrical Anisotropy
If some $\gamma(h)$ with different direction has the same sill $C$ and nugget variance, while the ranges $a$ are different, then they shows geometrical anisotropy.

In common the magnitude of range $a$ will be distributed following an ellipsoid. This phenomena could be occurred for placer deposit (tin, gold, iron ore, etc.).
A pattern of geometrical anisotropy (ellipsoid)

- \( a_{N-S} \) : range for \( N-S \) direction
- \( a_{NE-SW} \) : range for \( NE-SW \) direction
- \( a_{E-W} \) : range for \( E-W \) direction
- \( a_{NW-SE} \) : range for \( NW-SE \) direction
Zonal Anisotropy

In some cases, the variogram for certain direction is truly different with the others, i.e. variogram for bedding deposit (sedimentary structure), where grade variation perpendicular to the bedding structure (vertical direction) is so large compared to grade variation parallel to the bedding structure (horizontal direction).

In this case, variogram model is perfectly anisotropic and could be decomposed as:

Isotropic component:

$$\gamma(h) = \gamma_1 \left( \sqrt{h_1^2 + h_2^2 + h_3^2} \right)$$

True anisotropic component is obtained from the variogram in vertical direction $\gamma_2(h_3)$, so the equation is:

$$\gamma(h_1, h_2, h_3) = \gamma_1 \left( \sqrt{h_1^2 + h_2^2 + h_3^2} \right) + \gamma_2(h_3)$$
6. Proportional Effect

In some cases, the variances in an area depends on the local mean. This could be seen from the relationship between variance and the square of means, i.e. in drilling group data set.

Example: Relationship between variance ($\sigma^2$) and local mean ($Z^2$) for Mo deposit, and the variograms for each level with different values of $\gamma(\infty)$. 
If the relationship between variance and square of local mean is linear, then their relative variogram could be defined, i.e. each steps of experimental variogram calculation must be divided by the square of local mean:

\[
\gamma(h) = \frac{1}{2} \frac{\sum_{i=1}^{N(h)} [z(x_i) - z(x_{i+h})]^2 / N(h)}{[\bar{Z}(h)]^2}
\]

with

\[
\bar{Z}(h) = \frac{1}{2} \sum_{i=1}^{N(h)} \frac{[Z(x_i) - Z(x_{i+h})]}{2} / N(h)
\]

So the relative variogram could be obtained as below. The phenomena of proportionaal effect can be found commonly for data with lognormal distribution.
7. Drift

Drift is found for the variogram with normal behaviour in the beginning, i.e. it rises up to the sill, but abruptly it rises up again as parabolic. It means that the regionalized variables is not stationary anymore. The drift could be defined easily by calculating the mean difference of variables $x_i$ and $x_{i+h}$ according to their vector direction $h$:

$$\Delta(h) = \frac{1}{2} \sum_{i=1}^{N(h)} \left[ \frac{1}{2} [Z(x_i) - Z(x_{i+h})] \right] / N(h)$$

and then visualized as graph. If the drift is absent, then the value of $\Delta(h)$ will be scattered around the axis of $h$.

An example of parabolic effect of a drift of the variogram of: (A) sulphur of coal mine, and (B) lead of Pb-Zn mine.
8. Hole Effect

When the variogram is calculated along the data distribution with high values and then low values, i.e. grades of channel sampling across ore veins, then after reaching the sill, it will be rising up and going down periodically.

Below is an example of hole effect from Clark (*Practical Geostatistics*) and Journel & Huijbregts (*Mining Geostatistics*).
As well as the histogram which has mathematical model, i.e. normal distribution, etc., experimental variogram has mathematical (fitting) model which will be required for estimation.

The mathematical/fitting model is defined according to:

1. **Variogram behaviour near the origin**, which is commonly easy to be known. The presence or absence of nugget variance can be known by extrapolating $\gamma(h)$ across the vertical axis (for $h = 0$).
2. The presence of sill; initially the statistics variance of data can be assumed as the value of total sill.
3. **The presence of anisotropy, nested structure, drift, etc.**

Based on the presence or absence of the sill and range, variogram model is classified into: model with sill and model without sill.
1. Variogram Model with Sill (Bounded Model)

a. Linear near the origin: Spherical Model (Matheron Model)

\[ \gamma(h) = c_0 + c \left( \frac{3|h|}{2a} - \frac{1}{2}\left(\frac{|h|}{a}\right)^3 \right) \quad \text{for } h < a \]

\[ \gamma(h) = c_0 + c \quad \text{for } h \geq a \]

\[ \gamma(h) = 0 \quad \text{for } h = 0 \]

\( a = \text{range}, \ C = \text{sill} = \gamma(\infty) \)

\[
\gamma(h) = c_0 + c \left( \frac{3|h|}{2a} - \frac{1}{2}\left(\frac{|h|}{a}\right)^3 \right) \quad \text{for } h < a
\]

\[
\gamma(h) = c_0 + c \quad \text{for } h \geq a
\]

\[
\gamma(h) = 0 \quad \text{for } h = 0
\]

\( a = \text{range}, \ C = \text{sill} = \gamma(\infty) \)

Spherical Model Variogram
b. Linear near the origin: Exponential Model (Formery Model)

\[ \gamma(h) = C_0 + C \left[1 - e^{-|h|/a} \right] \quad \text{for } h < a \]

\(a\) = range which is an abscissa of crossing-point between tangential line of variogram and the sill (C).

Exponential Model Variogram
Fig. 3.10. (a) The spherical variogram model with a sill of 1.0 and a range of 1.0 and (b) an exponential model with a sill of 1.0 and a scale parameter of 0.33 (i.e. its practical range is 1.0)  
(Source: Armstrong, 1998)
c. Parabolic near the origin: Gaussian Model

\[ \gamma(h) = C_0 + C \left[ 1 - e^{-\left(\frac{|h|}{a}\right)^2} \right] \quad \text{for } h < a \]
2. Variogram Model without Sill (Unbounded Model)

Variogram model without sill includes:

a. Linear Model: $\gamma(h) = p|h|

Or in common known as Power Model: $\gamma(h) = p|h|^\lambda$

where: $p$ is a constant which is proportional to the absolute $h$, and $0 < \lambda < 2$, if $\lambda = 2$, then the model to be parabolic.
b. Logarithmic Model or de Wijsian Model: \( \gamma(h) = 3\alpha \log|\lambda h| + B \)

where: \( B = C_0 + 3\alpha(3/2 - \log l) \), with \( 3\alpha \) is coefficient of absolute dispersion and equal to the increasing of variogram if \( h \) is expressed to be logarithm, and \( l \) is equivalent length of sample.

Logarithmic Model Variogram
Simulation of a variable having a variograms of: (a) spherical, (b) exponential, (c) Gaussian, and (d) cardinal sine (from Armstrong, 1998)
3. Variogram Fitting

Experimental variogram is useful for structural analysis of a deposit, and it could not be used directly in reserve estimation. Therefore, the theoretical model of variogram is required to be fitted into the experimental variogram. The theoretical model is expressed by mathematical model.

The mathematical model of variogram which is commonly used for any types of ore deposits is Spherical Model or Matheron Model (David, 1977; Barnes, 1979). Therefore in experimental variogram fitting will be only discussed for Spherical Model.

There two common methods used for fitting of experimental variogram with its theoretical model: visual method and least square method. In common, the fitting result by visual (manual) method is quite sophisticated, and it has been used commonly by geostatistician (David, 1979). Due to the sense gave much essence in the fitting model, so this exercise needs much experiences in order to produce the high quality of fitting model.

The main objective of fitting model is: to define ”geostatistical parameters” → a, C and C₀.
A guidance in variogram fitting model:

1. Variogram with less number of pairs can be ignored.
2. Nugget variance \((C_0)\) can be obtained from the crossed tangential line of some first variogram points to the axis \(\gamma(h)\).
3. Sill \((C_0+C)\) is approximately equal or close to the population variance. Tangential line will across sill line in distance \(2/3a\), so the range \(a\) can be defined (David, 1977; Clark, 1979; Leigh & Readdy, 1982).
4. Interpretation on nugget variance for variogram with angle tolerance >90° (omnidirectional) will be helpful in anticipating the magnitude of nugget variance (David, 1979).
5. Variogram fitting must consider the experimental variogram near the origin, and then regards the variogram with high number of pairs.
Take care...nugget effect ! ! !

Nugget ratio = $\frac{C_0}{(C + C_0)} \times 100\%$

Low-nugget ratio $\rightarrow < 25\%$
Medium-nugget ratio $\rightarrow 25 - 50\%$
High-nugget ratio $\rightarrow 50 - 75\%$
Extreme-nugget ratio $\rightarrow > 75\%$

(Source: Dominy et al., 2003)
The existence of outliers in thickness M2 (a) & total sulfur M1 (b)

(Source: Heriawan et al., 2004)
Variograms of thickness M2 (left) & total sulfur M1 (right); (a) using all data, (b) without outliers

(Source: Heriawan et al., 2004)
Case: 3D variogram of Pb-Zn grades

(Source: Heriawan et al., 2008)
3D variogram of Pb grade (all data) fitted in SGeMS
Case: 3D Cu-Au Porphyritic Deposit
3D omnidirectional variogram for Au (ppm) in rocktype A

\[ \gamma(h) = 0.04 \text{Nug}(h) + 0.23 \text{Sph}(h/190) \]
3D omnidirectional variogram for Au (ppm) in rocktype B

\[ \gamma(h) = 0.007 \text{Nug}(h) + 0.02 \text{Sph}(h/500) \]
3D omnidirectional variogram for Ag (ppm) in all rocktypes (A+B)

\[ \gamma(h) = 0.45 \text{Nug}(h) + 0.95 \text{Sph}(h/145) \]
3D omnidirectional variogram for Cu (%) in all rocktypes (A+B)

\[ \gamma(h) = 0.01 \text{ Nug}(h) + 0.055 \text{ Sph}(h/370) \]
• The relation between sampling (drillhole) grade and block grade shows a systematical scattering.

• It means that drillhole sampling is not the best estimation for a block, so it is need a correction.

• Matheron (1962) introduced a correction by weighting the sampling values by means of variogram function.

• The name of ”kriging” adopted from the name of a mine engineer (statistician) from South Africa D.G. Krige who firstly thought about the matter since 1950.

• The correlation between grade of drillhole samples and true grade of blocks which represented by the drillhole (obtained after mining the blocks) will give a scatter plot which shows that the most of data (points) situated within the ellipsoid as seen in the next figure.
Data scattering between grade of drillhole samples vs. grade of mining blocks

- In case the grade analysis of samples is the right estimation to the grade of each blocks, then the regression through the origin will be along line A-A'.
- The research from Krige for Au grade samples showed that in reality the slope of regression line was a bit low as seen by line B-B' (the next figure).
This means that the deviation is systematic and drillhole samples are not the representative values for the blocks grade.

Samples grade higher than average value gives the higher value into the blocks grade, if not corrected.

For example: Sample grade \( z_1 \) gives block grade \( Z_1' \) by line A-A’ which is higher than the true block grade \( Z_1 \) (line B-B’).
• On the other hand, the sample grade lower the average value gives the value lower than block grade, for example: sample \( z_2 \) by line A-A’ gives block grade \( Z_2' \) which is lower than the true block grade \( Z_2 \) (line B-B’).

• Matheron’s correction by considering the variogram of regional data analysis showed that estimation on blocks grade was not only influenced by the samples within the blocks but also influenced by the samples around them in vicinity.

• The correction gives:
  - better estimated values,
  - variance of estimation \( \sigma_k^2 \)

• Estimation by Kriging method sometimes is too complex for a commodity (related to geometrical parameters). But it will be so useful to define the mineable reserve with grades above cut-off grade.

• As for example: correlation between samples grade and blocks grade (true grade) scattered within the ellipsoid (the next figure), then giving a regression line through the origin (0) and point \( (\bar{Z}, \bar{z}) \). Finally, the ellipsoid can be divided into four sectors by the cut-off grade (cog), \( z_c = Z_c = 5\% \).
Data scattering between samples grade vs. Blocks grade which showed mistaken mining

- **Sector 1**: all blocks with grade \( > cog \) coincides to samples grade \( > cog \) \( \rightarrow \) have been mined
- **Sector 2**: all blocks with grade \( < cog \) coincides to the samples grade \( < cog \) \( \rightarrow \) have not been mined
- **Sector 3**: all blocks with grade \( < cog \) due to the mistaken samples grade \( > cog \) \( \rightarrow \) have been mined
- **Sector 4**: all blocks with grade \( > cog \) due to the mistaken samples grade \( < cog \) \( \rightarrow \) have not been mined
• If the regression line \( B-B' \) which shows the correlation between samples and blocks grade is plotted, then blocks with grade 5% is mineable although the sample grade is 3.5% (Sector 1 in Fig. b).

• Sector 4 in Fig. b which is not mineable due to the mistaken information will be smaller, while Sector 3 which is mineable although the samples grade is low will be larger. But in general, blocks with grade > cut-off grade (5%) and mineable will be larger (Sector 1).

• Based on variogram analysis, Matheron gave correction on estimated grade on blocks which not considering only the sample within blocks, but also considering the samples around them.

• By means of Kriging method, we will not define the new better regression line, but this method will correct the samples grade to be higher or lower until the scattering within ellipsoid is tighter (see the next figure).
The change of ellipsoid of data scattering after correction by Kriging method

- By the correction using Kriging method, the shape of ellipsoid will be thinner/tighter with the border closer to the regression line with tangential 45°.
- The number of samples and associated blocks in Sectors 3 and 4 which indicates mineable low grade or not mineable high grade will be less smaller.
- Royle & Newton (1972) searched all kind of correction model and concluded that Kriging process gave the best estimated values based on the corrected samples grade.
**General Formula**

Assume we have a group $S_1$ of $n$ samples with the same volume in a place $x_i$. Estimation of grade $Z$ from the volume $V$ is $Z^*$. This estimated value obtained by the weighting on samples grade $z(x_i)$:

$$ Z^* = \sum_{i=1}^{n} \lambda_i \cdot z(x_i) $$

The sum of weights factor is set to be 1: $\sum_{i=1}^{n} \lambda_i = 1$

By this way we will get estimated values without biased (unbiased), which means the difference between $Z$ and $Z^*$ insist to be 0.

$$ E\{Z - Z^*\} = 0 $$

By considering the weighting factor, we will get an estimation variance as:

$$ \sigma^2_E = \text{Var} [Z - Z^*] $$

$$ = \frac{2}{V} \sum_{i=1}^{n} \lambda_i \int_{V} \gamma(x_i - y)dy - \frac{1}{VV} \int_{V} \int_{V} \gamma(x - y)dx dy - \sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_i \lambda_j \gamma(x_i - x_j) $$

$$ = 2 \sum_{i=1}^{n} \lambda_i \bar{\gamma}(S_i, V) - \bar{\gamma}(V, V) - \sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_i \lambda_j \gamma(S_i, S_j) $$
Estimation variance is the function of weighting factors which is known their sum is 1. To choose the optimum weighting factors, the estimation variance is set to be minimum.

Requirement that the sum of unknown $\lambda_i$ is 1 can be approximated by the help of Lagrange multiplier ($\mu$) to minimize the following equation:

$$Q = \sigma_E^2 - 2\mu\left(\sum \lambda_i - 1\right) \rightarrow \text{minimum}$$

Beside the unknown $\lambda_i$, the $\mu$ is also unknown. The minimizing above formula means that the partial of $\partial Q/\partial \mu$ and $\partial Q/\partial \lambda_i$ is equal to 0.

Finally we get the linear equation system of Ordinary Kriging (OK):

$$\sum_{j=1}^{n} \lambda_j \gamma(x_i - x_j) + \mu = \frac{1}{V} \int_V \gamma(x-x_i) dx \quad \text{or} \quad \sum_{j=1}^{n} \lambda_j \gamma(x_i - x_j) + \mu = \bar{\gamma}(x_i, V) \quad \text{and} \quad \sum_{i=1}^{n} \lambda_i = 1$$

This system is used to define the values of $\lambda_i$ and $\mu$ which will generate the minimum estimation variance.

Kriging known as BLUE = “Best Linear Unbiased Estimator”
Estimation variance (OK variance) is expressed by:

\[
\sigma_{OK}^2 = \frac{1}{VV} \int \int \int \gamma(x-y) dy - \sum_{j=1}^{n} \lambda_j \frac{1}{V} \int \gamma(x-x_j) dx \quad \text{or}
\]

\[
\sigma_{OK}^2 = -\bar{\gamma}(V,V) + \mu + \sum_{j=1}^{n} \lambda_j \bar{\gamma}(s_j, V)
\]

Ordinary Kriging (OK) is used when the local mean of regionalized variable is unknown. In case the local mean of regionalized variable is known (i.e. case of perfectly stationary data) as \(m\), then Simple Kriging (SK) should be applied.

**Simple Kriging System:**

\[
\sum_{j=1}^{n} \lambda_j C(x_i, x_j) = \bar{C}(x_i, V) \quad i = 1, 2, \ldots, N
\]

\[
\sigma_{SK}^2 = \bar{C}(V, V) - \sum_{i=1}^{n} \lambda_i \bar{C}(x_i, V)
\]

\[
Z^* = \sum_{i=1}^{n} \lambda_i \cdot z(x_i) + m \lambda_m \quad \Rightarrow \quad \lambda_m = 1 - \sum_{i=1}^{n} \lambda_i
\]
Which is the most unbiased estimation method?
The Effect of Geostatistical Parameters to the Weighting Factors and Estimation Variance

Effects of geostatistical parameters will be explained by some simple example below:

Known samples $x_i$ with grades $z(x_i)$ were taken with same distance ($L=20$ m) along the line. Variogram (Matheron model) of the data has parameters: $C_0 = 0$; $C = 1$; $a = 60$ m.

- We will calculate the weighting factors, estimation variance (kriging variance), and relative standard deviation for grade $\bar{z}^*$ along the line $L$ (i.e. on point $x_1$).
- In order to know how the effect of aureole samples and nugget variance, we will consider the influence of: (i) one point ($x_1$ itself), (ii) three points ($x_1, x_2, x_3$), and (iii) all points.
## Summary

<table>
<thead>
<tr>
<th>Parameters</th>
<th>1 Sample</th>
<th>3 Samples</th>
<th>All Samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>C₀</td>
<td>0.0</td>
<td>0.3</td>
<td>0.5</td>
</tr>
<tr>
<td>λ₁</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>λ₂</td>
<td>0.12</td>
<td>0.22</td>
<td>0.24</td>
</tr>
<tr>
<td>λ₃</td>
<td>0.12</td>
<td>0.22</td>
<td>0.24</td>
</tr>
<tr>
<td>σ²ₖ</td>
<td>0.08</td>
<td>0.38</td>
<td>0.58</td>
</tr>
<tr>
<td>σₖ</td>
<td>29%</td>
<td>62%</td>
<td>76%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>3 Samples</th>
<th>All Samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>C₀</td>
<td>0.0</td>
<td>0.3</td>
<td>0.5</td>
</tr>
<tr>
<td>λ₁</td>
<td>0.76</td>
<td>0.57</td>
<td>0.51</td>
</tr>
<tr>
<td>λ₂</td>
<td>0.12</td>
<td>0.22</td>
<td>0.24</td>
</tr>
<tr>
<td>λ₃</td>
<td>0.12</td>
<td>0.22</td>
<td>0.24</td>
</tr>
<tr>
<td>σ²ₖ</td>
<td>0.05</td>
<td>0.197</td>
<td>0.276</td>
</tr>
<tr>
<td>σₖ</td>
<td>23%</td>
<td>44%</td>
<td>53%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>0.0</th>
<th>0.3</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>C₀</td>
<td>0.0</td>
<td>0.3</td>
<td>0.5</td>
</tr>
<tr>
<td>λ₁</td>
<td>0.76</td>
<td>0.54</td>
<td>0.47</td>
</tr>
<tr>
<td>λ₂</td>
<td>0.22</td>
<td>0.34</td>
<td>0.34</td>
</tr>
<tr>
<td>λ₃</td>
<td>0.22</td>
<td>0.12</td>
<td>0.25</td>
</tr>
<tr>
<td>σ²ₖ</td>
<td>0.05</td>
<td>0.19</td>
<td>0.25</td>
</tr>
<tr>
<td>σₖ</td>
<td>23%</td>
<td>43%</td>
<td>50%</td>
</tr>
</tbody>
</table>
Properties of Kriging Method

By means of Kriging method we obtain the best estimator based on the available information of a mineral deposit. The weight factors is chosen to obtain the minimum estimation variance.

So the Kriging considers about:

1. Structural and spatial correlation by means of function $\gamma(h)$.

2. Relative geometrical relation among estimator data and volume by means of $\gamma(s_i,s_j)$ (relation among data) and $\gamma(s_i,V)$ (relation between data and volume).

   • If the variogram is isotropic and data is regular, then kriging system will give symmetrical data.

   • In many cases, samples inside and around the estimated block give estimation, while sample far from the estimated block will have weights close to zero. In this case the searching radius will not have influence (screened).

   • The screen effect will occur, if the nugget effect is zero or very small $\varepsilon = C_o / C$. The nugget effect can reduce the screen effect. For the large nugget effect, all sample will be considered having the same weight.
The case of Kriging on Regular Grid

The calculation is performed for a block of mineral deposit with known variogram parameter with Matheron model and $C_0 = 0$; $C = 1$; $a = 60$ m.

Block is rectangular with size $20$ m $\times$ $30$ m and there are 4 sample around it and 1 sample in the middle.

Based on the symmetrical position of samples to block, then estimator formula is written as:

$$z^* = \lambda_1 \cdot z(x_1) + \lambda_2 \cdot \frac{z(x_2) + z(x_3)}{2} + \lambda_3 \cdot \frac{z(x_4) + z(x_5)}{2}$$
A slice of mineral deposit is known having block size 100 × 100 ft.

On each block, the sample was taken randomly (random stratified grid).

Variogram of data samples gives Matheron model with parameters:

\[ C = 16.50\%^2; \quad C_0 = 3.80\%^2; \quad \varepsilon = 0.23; \quad a = 240 \text{ ft; } \bar{z} = 4.27\% \]

To correct the samples values by considering block grades around them, kriging is indispensable.

The calculation is performed with minimum neighborhood (within searching radius) is 5 samples.

<table>
<thead>
<tr>
<th>Sample Grade</th>
<th>Case 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.75</td>
<td></td>
</tr>
<tr>
<td>4.27</td>
<td></td>
</tr>
<tr>
<td>4.14</td>
<td></td>
</tr>
<tr>
<td>3.76</td>
<td>4.87</td>
</tr>
<tr>
<td>2.61</td>
<td>3.51</td>
</tr>
<tr>
<td>8.46</td>
<td>5.51</td>
</tr>
<tr>
<td>0.02</td>
<td>1.96</td>
</tr>
<tr>
<td>1.30</td>
<td></td>
</tr>
<tr>
<td>4.57</td>
<td>4.48</td>
</tr>
<tr>
<td>5.10</td>
<td>3.09</td>
</tr>
<tr>
<td>2.12</td>
<td>4.88</td>
</tr>
<tr>
<td>1.93</td>
<td>2.91</td>
</tr>
<tr>
<td>5.40</td>
<td>5.37</td>
</tr>
<tr>
<td>5.19</td>
<td>4.14</td>
</tr>
<tr>
<td>0.17</td>
<td>1.96</td>
</tr>
<tr>
<td>3.20</td>
<td>2.77</td>
</tr>
<tr>
<td>2.39</td>
<td></td>
</tr>
<tr>
<td>4.85</td>
<td>4.65</td>
</tr>
<tr>
<td>9.06</td>
<td>5.79</td>
</tr>
<tr>
<td>5.13</td>
<td>3.78</td>
</tr>
<tr>
<td>0.43</td>
<td>1.52</td>
</tr>
<tr>
<td>1.52</td>
<td></td>
</tr>
<tr>
<td>0.92</td>
<td>2.03</td>
</tr>
<tr>
<td>10.80</td>
<td>2.68</td>
</tr>
<tr>
<td>7.31</td>
<td>3.56</td>
</tr>
<tr>
<td>2.68</td>
<td>1.75</td>
</tr>
<tr>
<td>1.96</td>
<td>2.26</td>
</tr>
<tr>
<td>4.12</td>
<td>5.80</td>
</tr>
<tr>
<td>5.67</td>
<td>9.20</td>
</tr>
<tr>
<td>8.95</td>
<td>26.40</td>
</tr>
<tr>
<td>15.51</td>
<td>9.17</td>
</tr>
<tr>
<td>9.09</td>
<td>5.91</td>
</tr>
<tr>
<td>5.59</td>
<td>2.51</td>
</tr>
<tr>
<td>3.04</td>
<td></td>
</tr>
<tr>
<td>3.01</td>
<td>3.52</td>
</tr>
<tr>
<td>3.13</td>
<td>5.37</td>
</tr>
<tr>
<td>6.71</td>
<td>7.87</td>
</tr>
<tr>
<td>16.30</td>
<td>9.11</td>
</tr>
<tr>
<td>10.91</td>
<td>0.91</td>
</tr>
<tr>
<td>3.80</td>
<td>3.12</td>
</tr>
<tr>
<td>3.37</td>
<td></td>
</tr>
<tr>
<td>0.88</td>
<td>1.81</td>
</tr>
<tr>
<td>0.12</td>
<td>2.82</td>
</tr>
<tr>
<td>1.62</td>
<td>3.72</td>
</tr>
<tr>
<td>3.60</td>
<td>3.78</td>
</tr>
<tr>
<td>3.46</td>
<td>4.29</td>
</tr>
<tr>
<td>3.62</td>
<td>3.59</td>
</tr>
<tr>
<td>1.49</td>
<td>2.00</td>
</tr>
<tr>
<td>2.71</td>
<td>2.12</td>
</tr>
<tr>
<td>2.93</td>
<td>3.28</td>
</tr>
<tr>
<td>4.04</td>
<td>3.35</td>
</tr>
<tr>
<td>5.06</td>
<td>4.47</td>
</tr>
<tr>
<td>0.19</td>
<td>2.91</td>
</tr>
<tr>
<td>2.63</td>
<td>3.28</td>
</tr>
<tr>
<td>2.68</td>
<td>8.25</td>
</tr>
<tr>
<td>5.63</td>
<td>1.78</td>
</tr>
</tbody>
</table>
Estimator: $z^* = \lambda_1 z_1 + \lambda_2 z_2 + \lambda_3 z_3$

with: $\lambda_3 = 1 - \lambda_1 - \lambda_2$

$z_1 =$ sample grade in the middle of block

$z_2 =$ samples grades of 5 to 8 (in blocks around)

$z_3 = z =$ average grades of all samples

Variance of estimation depends on the number of samples involved in estimation:

<table>
<thead>
<tr>
<th>Nb. of sample within block</th>
<th>Aureoles</th>
<th>Variance</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>3,68</td>
<td>1,9</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
<td>3,99</td>
<td>2,0</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>4,25</td>
<td>2,1</td>
</tr>
<tr>
<td>0</td>
<td>6</td>
<td>8,43</td>
<td>2,9</td>
</tr>
</tbody>
</table>

Previous map/figure shows sample values (written as large font) and kriged values below it (written as italic small font)
According to the configuration of blocks and searching radius of kriging with 5 samples in minimum, then there are 78 kriged blocks from the total of 88 blocks. The cut-off grade is defined as 3.0%.

**Problems:**

1. How many parts of 78 kriged blocks which have original sample grades >3.0%?
2. How many parts of 78 kriged blocks which have kriged grades >3.0%?
3. Please remark the blocks with kriged grades >3.0% as economics blocks!

**References**

<table>
<thead>
<tr>
<th>Original Samples</th>
<th>Estimated Blocks (Case 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>2.75  4.27  2.01</td>
<td>3.76  2.61  8.46  0.02  1.30</td>
</tr>
<tr>
<td>4.57  5.10  2.12  4.88  1.93</td>
<td>4.85  9.06  5.79  3.67  0.43  1.52</td>
</tr>
<tr>
<td>5.40  5.37  3.57  0.17  3.20  2.39</td>
<td>5.37  5.19  4.14  1.96  2.77</td>
</tr>
<tr>
<td>4.85  9.06  5.79  3.67  0.43  1.52</td>
<td>4.65  6.57  5.13  3.78  1.57  2.05</td>
</tr>
<tr>
<td>0.92  2.03  10.80  2.68  1.75  1.33</td>
<td>4.00  7.31  3.56  2.68  1.96  2.26</td>
</tr>
<tr>
<td>7.47  1.54  0.45  2.68  6.93  4.40</td>
<td>5.53  3.73  5.37  4.32  1.91  2.92  5.22</td>
</tr>
<tr>
<td>2.23  12.31  6.65  2.34  1.92  5.79  4.79</td>
<td>4.27  8.74  6.27  3.49  3.27  4.69  4.66</td>
</tr>
<tr>
<td>3.50  10.01  7.50  7.15  4.61  3.63  2.56</td>
<td>4.85  7.90  9.03  7.98  6.53  4.24  3.54</td>
</tr>
<tr>
<td>4.12  5.80  9.20  28.40  9.17  5.91  2.51</td>
<td>5.67  8.95  15.51  9.09  5.59  3.04</td>
</tr>
<tr>
<td>3.01  3.13  6.71  16.30  0.91  3.12</td>
<td>3.52  5.37  7.87  10.91  3.80  3.37</td>
</tr>
<tr>
<td>0.88  0.12  2.82  1.62  3.78  3.62</td>
<td>1.81  3.72  3.60  4.29  3.59</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>1.49  2.71  2.93  3.46  5.06</td>
<td>2.00  2.12  2.89  3.35  4.47</td>
</tr>
<tr>
<td>1.02  2.77  1.22  4.72  3.33</td>
<td>1.88  2.23  2.75  4.04  3.95</td>
</tr>
<tr>
<td>0.19  2.91  2.68  8.75  1.78</td>
<td>2.63  3.39  5.63</td>
</tr>
</tbody>
</table>
The same mineral deposit is calculated again using Kriging by assuming that all drillholes exactly located in the middle of grid/block.

The kriging result can be shown in the figure beside.

<table>
<thead>
<tr>
<th></th>
<th>sample grade</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.75</td>
<td>4.27</td>
<td>2.01</td>
</tr>
<tr>
<td>3.76</td>
<td>8.46</td>
<td>0.02</td>
</tr>
<tr>
<td>4.57</td>
<td>2.12</td>
<td>4.88</td>
</tr>
<tr>
<td>5.40</td>
<td>0.17</td>
<td>3.20</td>
</tr>
<tr>
<td>4.85</td>
<td>5.79</td>
<td>3.67</td>
</tr>
<tr>
<td>0.92</td>
<td>10.80</td>
<td>2.68</td>
</tr>
<tr>
<td>7.47</td>
<td>1.54</td>
<td>1.75</td>
</tr>
<tr>
<td>4.12</td>
<td>5.80</td>
<td>2.68</td>
</tr>
<tr>
<td>3.50</td>
<td>10.01</td>
<td>7.15</td>
</tr>
<tr>
<td>4.12</td>
<td>5.80</td>
<td>9.20</td>
</tr>
<tr>
<td>3.01</td>
<td>3.13</td>
<td>0.88</td>
</tr>
<tr>
<td>0.12</td>
<td>1.30</td>
<td>1.49</td>
</tr>
<tr>
<td>0.19</td>
<td>2.91</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Kriged blocks with max and min samples of 9 and 6 samples respectively.
### Estimated Blocks (Case 1)

<p>| | | | | | | | | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3.87</td>
<td>3.51</td>
<td>5.51</td>
<td>1.96</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.48</td>
<td>4.74</td>
<td>3.09</td>
<td>3.51</td>
<td>2.31</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.37</td>
<td>5.19</td>
<td>4.14</td>
<td>1.96</td>
<td>2.77</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.65</td>
<td>6.57</td>
<td>5.13</td>
<td>3.78</td>
<td>1.57</td>
<td>2.05</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.00</td>
<td>7.31</td>
<td>3.56</td>
<td>2.68</td>
<td>1.96</td>
<td>2.26</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.53</td>
<td>3.73</td>
<td>5.37</td>
<td>4.32</td>
<td>1.91</td>
<td>2.92</td>
<td>5.22</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.27</td>
<td>8.74</td>
<td>6.27</td>
<td>3.49</td>
<td>3.27</td>
<td>4.69</td>
<td>4.66</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.85</td>
<td>7.90</td>
<td>9.03</td>
<td>7.98</td>
<td>6.53</td>
<td>4.24</td>
<td>3.54</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.67</td>
<td>8.95</td>
<td>15.51</td>
<td>9.09</td>
<td>5.59</td>
<td>3.04</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.52</td>
<td>5.37</td>
<td>7.87</td>
<td>10.91</td>
<td>3.80</td>
<td>3.37</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.81</td>
<td>3.72</td>
<td>3.60</td>
<td>4.29</td>
<td>3.59</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.00</td>
<td>2.12</td>
<td>2.89</td>
<td>3.35</td>
<td>4.47</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.88</td>
<td>2.23</td>
<td>2.75</td>
<td>4.04</td>
<td>3.95</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.63</td>
<td>3.39</td>
<td>5.63</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Estimated Blocks (Case 2)

<p>| | | | | | | | | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3.62</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.66</td>
<td>3.91</td>
<td>4.99</td>
<td>1.86</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.59</td>
<td>4.46</td>
<td>3.67</td>
<td>3.04</td>
<td>2.09</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.30</td>
<td>5.48</td>
<td>3.64</td>
<td>2.05</td>
<td>2.29</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.73</td>
<td>6.65</td>
<td>5.98</td>
<td>3.13</td>
<td>1.41</td>
<td>1.38</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.51</td>
<td>6.82</td>
<td>3.63</td>
<td>1.19</td>
<td>0.64</td>
<td>1.66</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.23</td>
<td>4.18</td>
<td>5.62</td>
<td>3.97</td>
<td>1.38</td>
<td>2.70</td>
<td>5.04</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.15</td>
<td>8.88</td>
<td>6.72</td>
<td>3.31</td>
<td>3.02</td>
<td>4.93</td>
<td>4.53</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.52</td>
<td>8.39</td>
<td>8.18</td>
<td>8.53</td>
<td>5.45</td>
<td>4.21</td>
<td>3.24</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.67</td>
<td>8.95</td>
<td>15.51</td>
<td>9.09</td>
<td>5.59</td>
<td>3.04</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.06</td>
<td>9.69</td>
<td>16.47</td>
<td>10.71</td>
<td>5.12</td>
<td>2.98</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.24</td>
<td>4.35</td>
<td>9.08</td>
<td>10.39</td>
<td>4.11</td>
<td>2.97</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.30</td>
<td>3.05</td>
<td>3.99</td>
<td>3.39</td>
<td>3.69</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.49</td>
<td>2.48</td>
<td>2.67</td>
<td>3.68</td>
<td>4.33</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.29</td>
<td>2.32</td>
<td>2.42</td>
<td>4.29</td>
<td>3.69</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.32</td>
<td>3.48</td>
<td>5.58</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Error standard deviation (Case 1: Random stratified grid)

<table>
<thead>
<tr>
<th>1500</th>
<th>1400</th>
<th>1300</th>
<th>1200</th>
<th>1100</th>
<th>1000</th>
<th>900</th>
<th>800</th>
<th>700</th>
<th>600</th>
<th>500</th>
<th>400</th>
<th>300</th>
<th>200</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.75</td>
<td>1.07</td>
<td>1.97</td>
<td>2.61</td>
<td>3.38</td>
<td>4.22</td>
<td>5.10</td>
<td>6.06</td>
<td>7.08</td>
<td>8.09</td>
<td>9.12</td>
<td>10.32</td>
<td>12.55</td>
<td>15.37</td>
<td></td>
</tr>
<tr>
<td>1.20</td>
<td>1.45</td>
<td>1.61</td>
<td>1.79</td>
<td>1.92</td>
<td>2.42</td>
<td>2.81</td>
<td>3.27</td>
<td>3.80</td>
<td>4.38</td>
<td>5.02</td>
<td>5.94</td>
<td>7.15</td>
<td>8.46</td>
<td></td>
</tr>
<tr>
<td>1.20</td>
<td>1.26</td>
<td>2.03</td>
<td>3.67</td>
<td>3.72</td>
<td>5.02</td>
<td>5.90</td>
<td>6.34</td>
<td>7.07</td>
<td>8.09</td>
<td>9.15</td>
<td>10.31</td>
<td>12.67</td>
<td>15.48</td>
<td></td>
</tr>
<tr>
<td>1.20</td>
<td>2.00</td>
<td>3.40</td>
<td>5.01</td>
<td>5.03</td>
<td>6.87</td>
<td>8.37</td>
<td>9.71</td>
<td>11.15</td>
<td>12.70</td>
<td>14.31</td>
<td>16.77</td>
<td>20.33</td>
<td>24.99</td>
<td></td>
</tr>
<tr>
<td>1.20</td>
<td>2.00</td>
<td>3.40</td>
<td>5.01</td>
<td>5.03</td>
<td>6.87</td>
<td>8.37</td>
<td>9.71</td>
<td>11.15</td>
<td>12.70</td>
<td>14.31</td>
<td>16.77</td>
<td>20.33</td>
<td>24.99</td>
<td></td>
</tr>
</tbody>
</table>

### Error standard deviation (Case 2: Regular grid)

<table>
<thead>
<tr>
<th>1500</th>
<th>1400</th>
<th>1300</th>
<th>1200</th>
<th>1100</th>
<th>1000</th>
<th>900</th>
<th>800</th>
<th>700</th>
<th>600</th>
<th>500</th>
<th>400</th>
<th>300</th>
<th>200</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.75</td>
<td>3.76</td>
<td>4.57</td>
<td>5.4</td>
<td>4.85</td>
<td>1.33</td>
<td>1.05</td>
<td>1.05</td>
<td>1.24</td>
<td>1.33</td>
<td>1.40</td>
<td>1.52</td>
<td>1.31</td>
<td>1.28</td>
<td></td>
</tr>
<tr>
<td>1.06</td>
<td>1.33</td>
<td>1.07</td>
<td>1.04</td>
<td>1.31</td>
<td>1.05</td>
<td>1.05</td>
<td>1.24</td>
<td>1.33</td>
<td>1.40</td>
<td>1.52</td>
<td>1.31</td>
<td>1.28</td>
<td>1.05</td>
<td></td>
</tr>
<tr>
<td>0.94</td>
<td>0.85</td>
<td>0.84</td>
<td>0.85</td>
<td>0.86</td>
<td>0.86</td>
<td>0.86</td>
<td>0.86</td>
<td>0.86</td>
<td>0.86</td>
<td>0.86</td>
<td>0.86</td>
<td>0.86</td>
<td>0.86</td>
<td></td>
</tr>
<tr>
<td>1.33</td>
<td>1.28</td>
<td>1.40</td>
<td>1.10</td>
<td>1.33</td>
<td>1.05</td>
<td>1.05</td>
<td>1.24</td>
<td>1.33</td>
<td>1.40</td>
<td>1.52</td>
<td>1.31</td>
<td>1.28</td>
<td>1.05</td>
<td></td>
</tr>
<tr>
<td>0.94</td>
<td>0.85</td>
<td>0.84</td>
<td>0.85</td>
<td>0.86</td>
<td>0.86</td>
<td>0.86</td>
<td>0.86</td>
<td>0.86</td>
<td>0.86</td>
<td>0.86</td>
<td>0.86</td>
<td>0.86</td>
<td>0.86</td>
<td></td>
</tr>
</tbody>
</table>

**Note:** The tables above show the error standard deviation for two cases: (1) Random stratified grid and (2) Regular grid.
## Statistical Summary of Original Sample and Estimated Values

<table>
<thead>
<tr>
<th>Statistics of Samples Value</th>
<th>Statistics of Estimated Value (Case 1)</th>
<th>Statistics of Estimated Value (Case 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>Mean</td>
<td>Mean</td>
</tr>
<tr>
<td>4.24</td>
<td>4.51</td>
<td>4.40</td>
</tr>
<tr>
<td>Median</td>
<td>Median</td>
<td>Median</td>
</tr>
<tr>
<td>3.20</td>
<td>3.98</td>
<td>3.94</td>
</tr>
<tr>
<td>Mode</td>
<td>Mode</td>
<td>Mode</td>
</tr>
<tr>
<td>2.68</td>
<td>5.37</td>
<td>3.24</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>Standard Deviation</td>
<td>Standard Deviation</td>
</tr>
<tr>
<td>3.83</td>
<td>2.35</td>
<td>2.60</td>
</tr>
<tr>
<td>Coefficient of Variation</td>
<td>Coefficient of Variation</td>
<td>Coefficient of Variation</td>
</tr>
<tr>
<td>0.90</td>
<td>0.52</td>
<td>0.59</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>Kurtosis</td>
<td>Kurtosis</td>
</tr>
<tr>
<td>13.29</td>
<td>5.60</td>
<td>5.36</td>
</tr>
<tr>
<td>Skewness</td>
<td>Skewness</td>
<td>Skewness</td>
</tr>
<tr>
<td>2.92</td>
<td>1.90</td>
<td>1.83</td>
</tr>
<tr>
<td>Minimum</td>
<td>Minimum</td>
<td>Minimum</td>
</tr>
<tr>
<td>0.02</td>
<td>1.57</td>
<td>0.64</td>
</tr>
<tr>
<td>Maximum</td>
<td>Maximum</td>
<td>Maximum</td>
</tr>
<tr>
<td>26.40</td>
<td>15.51</td>
<td>16.47</td>
</tr>
<tr>
<td>Count</td>
<td>Count</td>
<td>Count</td>
</tr>
<tr>
<td>85.00</td>
<td>78.00</td>
<td>78.00</td>
</tr>
</tbody>
</table>
Evaluation:

• Estimation variance (\( \sigma_k^2 \)) is lower than before.

• Depending on the drillhole configuration and the number of \( N \) drillholes, the estimated values are different.

• The figure beside shows the properties of estimation variance and estimated values \( Z^* \) in relation to the number of drillholes \( N \) with 2 different patterns.

• It seems that 5 - 6 drillholes are quite enough for estimation.

The effect of pattern and number of samples to the kriging variance and average values
Point/Punctual Kriging Method

- In general samples points are distributed irregularly, so in order to arrange the isoline map, the interpolation for creating a regular grid is indispensable.

- Interpolation can be performed by some methods i.e. NNP (Nearest Neighboring Polygon) and IDW (Inverse Distance Weighted, ID, IDS, or ID3).

- From some discussions about estimation methods, Kriging has been known producing best and reliable estimator.

- In order to solve the estimation in point, the same kriging system previously is used. In this case, we use only variogram, because only the relation among points is considered.

- As for example (see the next figure): if there are 3 points \( \mathbf{x}_i \) to estimate the 4th point \( \mathbf{x}_0 \), the Kriging system written as:

\[
\begin{align*}
\lambda_1 \cdot \gamma(x_1, x_1) + \lambda_2 \cdot \gamma(x_1, x_2) + \lambda_3 \cdot \gamma(x_1, x_3) + \mu &= \gamma(x_1, x_0) \\
\lambda_1 \cdot \gamma(x_2, x_1) + \lambda_2 \cdot \gamma(x_2, x_2) + \lambda_3 \cdot \gamma(x_2, x_3) + \mu &= \gamma(x_2, x_0) \\
\lambda_1 \cdot \gamma(x_3, x_1) + \lambda_2 \cdot \gamma(x_3, x_2) + \lambda_3 \cdot \gamma(x_3, x_3) + \mu &= \gamma(x_3, x_0) \\
\lambda_1 + \lambda_2 + \lambda_3 &= 1
\end{align*}
\]
Kriging system solution and data configuration:

\[
\begin{align*}
\bar{\gamma}(x_1 x_1) &= \bar{\gamma}(x_2 x_2) = \bar{\gamma}(x_3 x_3) = 0,0, \\
\bar{\gamma}(x_1 x_2) &= \bar{\gamma}(x_2 x_1) = C_0 + C \cdot \gamma(x_1 - x_2) \\
\bar{\gamma}(x_1 x_3) &= \bar{\gamma}(x_3 x_1) = C_0 + C \cdot \gamma(x_1 - x_3) \\
\bar{\gamma}(x_2 x_3) &= \bar{\gamma}(x_3 x_2) = C_0 + C \cdot \gamma(x_2 - x_3) \\
\bar{\gamma}(x_1 x_0) &= C_0 + C \cdot \gamma(x_1 - x_0) \\
\bar{\gamma}(x_2 x_0) &= C_0 + C \cdot \gamma(x_2 - x_0) \\
\bar{\gamma}(x_3 x_0) &= C_0 + C \cdot \gamma(x_3 - x_0) 
\end{align*}
\]

Estimation/kriging variance is simplified to be:

\[
\sigma_K^2 = \mu + \sum_{j=1}^{n} \lambda_j \bar{\gamma}(x_j - x_0)
\]
Example of point kriging application by Delfiner & Delhomme (1973): *Optimum interpolation by kriging*, in Davies & McCullagh (Eds.), Display and Analysis of Spatial Data.

Linear variogram of rainfall data in Wadi Kadjemeur, Central Africa

Rainfall measurement points (mm), where contours constructed based on manual interpolation

Contours constructed based on polynomial order-2

Contours constructed based on point kriging method
BOX No 4: Steps in a case-study.

Step 1: Collect and check data.
If you were not actively involved from the start of the project, find those who were and ask them about
- the types of sampling and analyses used and any changes in procedure,
- different geological zones, faulting etc,
- preferential sampling, etc.
At the outset a series of major decisions has to be made.
- Whether to work with grades in 3D or with accumulations in 2D.
- The limits of area to study, the support of the variables and whether they are stationary,

Basic statistics (means, variances, correlations, histograms and scatter diagrams) are calculated. Look for
- outliers or abnormal values
- nonhomogeneous data (mixed populations)

Step 2: Calculate experimental variograms.
Step 3: Fit a variogram model.
Step 4: Kriging or simulation

(Source: Armstrong, 1998)
Spatial distribution of Ni+Co grades estimated using Ordinary Kriging (OK)

Sections of: (a) estimated Ni+Co grades, (b) kriging standard deviation, and (c) profile of original grades from test pit

Fence diagram in SW-NE direction

Source: Heriawan et al. (2004)
Application of Kriging using SGeMS (2004)
3D kriging estimation of Pb grade

Level 3025 m

Kriged Pb in HSZ (%)
3D kriging variance of Pb grade
To develop a quantitative approach, geostatistics were applied to the data on coal sulfur content from samples taken in the Pittsburgh coal bed.

Geostatistical methods that account for regional and local trends were applied to blocks 2.7 mi (4.3 km) on a side. The data and geostatistics support conclusions concerning the average sulfur content and its degree of reliability at regional- and economic-block scale.

To validate the method, a comparison was made with the sulfur contents in sample data taken from 53 coal mines located in the study area. The comparison showed a high degree of similarity between the sulfur content in the mine samples and the sulfur content represented by the geostatistically derived contours.
(a) Location of sulfur samples used to analyze the sulfur content of the Pittsburgh coal bed, and (b) Contours of estimated average sulfur content (wt.% ar) for blocks, 2.7 mi on a side (1 mi = 1.609 km).
Identifying Spatial Heterogeneity of Coal Resource Quality (Heriawan & Koike, 2008)

• A geostatistical approach was presented to characterization of the geometry and quality of a multilayer coal deposit in East Kalimantan, which associated with a synclinal structure.

• Semivariogram analysis clarified strong dependence of calorie on ash in top and bottom parts of each seam, and the existence of a strong correlation with sodium over the sub-seams in same location.

• Correlations between the geometry and quality of the seams were generally weak.

• Sodium shows distinct segregation: the low zones are concentrated near the boundary of sub-basin, while the high zones are concentrated in the central part.

• Geostatistical modeling results suggest that thicknesses of all major seams were controlled by syncline structure, while coal qualities chiefly were originated from the coal depositional and diagenetic processes.
Spatial distribution of coal qualities estimated using Ordinary Kriging (OK)

Seam thickness

Ash content

Total sulphur

Source: Heriawan & Koike (2008)
Application of Geostatistics for Resource Estimation
Area of Influence for Resources Classification (Case of Coal Deposit)

- USGS Circular 891 (0 - 400 m = measured, 400 - 1200 m = indicated, 1200 - 4800 m = inferred, > 4800 m = hypothetic)
- SNI 1999 (Indonesian Standard for Coal Resource Classification):

<table>
<thead>
<tr>
<th>Geological Condition</th>
<th>Criteria</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Measured</td>
</tr>
<tr>
<td>Simple</td>
<td>Information spacing (m)</td>
<td>X≤300</td>
</tr>
<tr>
<td>Moderate</td>
<td>Information spacing (m)</td>
<td>X≤200</td>
</tr>
<tr>
<td>Complex</td>
<td>Information spacing (m)</td>
<td>X≤100</td>
</tr>
</tbody>
</table>
General relationship between Exploration Results, Mineral Resources & Ore Reserves

Exploration Results

MINERAL RESOURCES

ORE RESERVES

Inferred

Indicated

Measured

Probable

Proved

Increasing level of geological knowledge and confidence

Consideration of mining, metallurgical, economic, marketing, legal, environmental, social and governmental factors (the “Modifying Factors”).

(Source: JORC, 2004)
### Subdivisions for Reserve Estimates (after Valee, 1986)

<table>
<thead>
<tr>
<th>Category</th>
<th>Data Condition</th>
<th>Approximate Margin of Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured ↔ Proven</td>
<td><strong>Developed:</strong> Ore or mineralization exposed and sampled in volume in addition to detailed drilling</td>
<td>0 – 10 %</td>
</tr>
<tr>
<td></td>
<td><strong>Drilled Defined:</strong> Ore or mineralization whose location, grade and continuity are established by regular and close-spaced drilling and sampling</td>
<td>5 – 20 %</td>
</tr>
<tr>
<td>Indicated ↔ Probable</td>
<td><strong>Class I:</strong> Ore or mineralization whose continuity and grade have been defined by regular, but fairly wide-spaced drilling or sampling</td>
<td>20 – 40 %</td>
</tr>
<tr>
<td></td>
<td><strong>Class II:</strong> Ore or mineralization whose continuity and grade have been defined by somewhat regular, wide-spaced drilling and sampling</td>
<td>40 – 70 %</td>
</tr>
<tr>
<td>Inferred ↔ Possible</td>
<td><strong>Potential Reserve:</strong> Mineralization interpreted on the basis of expected continuity from known exposures, and whose location and grade are only poorly surmised from a few irregular drillholes or exposures</td>
<td>70 – 100 %</td>
</tr>
</tbody>
</table>
Yamamoto (1999): The classification scheme proposed by Diehl and David (1982), with different confidence levels, is not acceptable, because ore-reserve estimation is carried out using a given exploration data. On the other hand, except for the use of OK standard deviations, the Wellmer’s ore-reserve classification seems to be reasonable. Note that this scheme uses a unique confidence level (90%), which is suitable for geological data as recommended by Koch and Link (1971).

References:
Normal distribution of estimation error
Blackwell (1998) demonstrated a practical use of relative-kriging variance (or relative-kriging standard deviation, RKSD) as an important component of resources classification scheme for porphyry Cu and large epithermal gold deposit:

1. Identify mineralized blocks (i.e. verify geologic continuity)
2. Identify mineralized blocks above cut-off grade.
3. Classify the blocks above cut-off grade based on the selected RKSD:

\[
\text{Measured } 0.3 \leq \text{Indicated } 0.5 \leq \text{Inferred}
\]
Case: Resources Classification of Ni Laterite in Different Block Area

South Block

Central Block

North Block
Case: Estimation on Cu porphyritic deposit using Ordinary Kriging (OK)
By assuming distribution of estimation error is normal with 95% of confidence level, relative error is calculated by estimation variance of Ordinary Kriging method:

\[
% \text{error} = \pm 1.96\sigma/z^* \times 100\%
\]

Based on % relative error as addressed in de Souza et al. (2002) which was modified from JORC 1999 and Diehl and David (1982), coal resources are classified to be:

1. Measured resources $\rightarrow$ relative error $\leq$ 10%
2. Indicated resources $\rightarrow$ relative error 10-20%
3. Inferred resources $\rightarrow$ relative error $> 20\%$
Estimated thickness (a) and its relative error (b); estimated sulphur content (c) and its relative error (d)
Resource Estimation – Simple Geological Condition
Resource Estimation – Moderate Geological Condition
Resource Estimation – Complex Geological Condition

Heriawan et al. (2010)
Coal Resource Estimation – Summary

Modified from Heriawan et al. (2010)
Some References


Thank You

Email: heriawan@mining.itb.ac.id